MAXIMA & MINIMA

EXERCISE – I   HINTS & SOLUTIONS

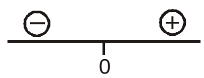
Sol.1 C

$f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + ... + 100x^{100}$

$f'(x) = 4x + 16x^3 + 36x^5 + ... + (100)^2 x^{99}$

$f'(x) = x(4 + 16x^2 + 36x^4 + ... + (100)^2 x^{98})$

$f'(x) = 0 \quad \Rightarrow \quad x = 0$



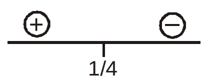
x = 0 is minima point

Sol.2 D

$f(x) = x^{25}(1 - x)^{75}$

$f'(x) = 25x^{24}(1 - x)^{75} - 75x^{25}(1 - x)^{74} = 0$

$\Rightarrow x = 1/4$



x = 1/4 maxima

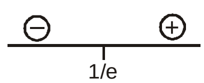
Sol.3 C

$f(x) = x^x$

$f'(x) = x^x(1 + \ell n\, x)$

$1 + \ell n\, x = 0$

$x = 1/e$

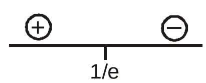


$1/e \rightarrow$ minima

$f(x) = x^{-x}$

$f'(x) = -x^{-x}(1 + \ell n\, x)$

$x = 1/e$



$1/e \rightarrow$ maxima

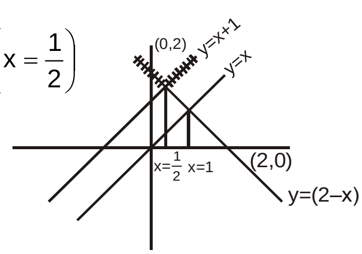
min. value $= \left(\dfrac{1}{e}\right)^{1/e}$ $\qquad f\left(\dfrac{1}{e}\right) = e^{1/e}$

product $= (e^{-1/e})(e)^{1/e} = 1$

Sol.4 D

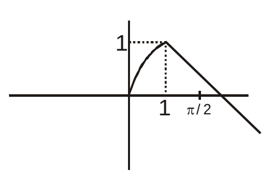
max. value at $\left(x = \dfrac{1}{2}\right)$

$= 1 + \dfrac{1}{2} = \dfrac{3}{2}$



Sol.5 A

x = 1 local maxima



Sol.6 C

$f(x) = 2 - \sqrt{1 + 2x + x^2} \qquad x \in [-2, 1]$

$f'(x) = \dfrac{2 + 2x}{2\sqrt{1 + 2x + x^2}} = 0 \quad \Rightarrow x = -1$

$f(1) = 2 - \sqrt{1 + 2 + 1} = 0$

$f(-1) = 2$

$f(-2) = 1$

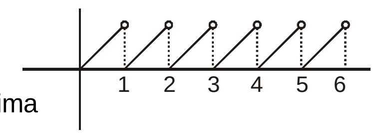
Greatest value = 2

least value = 0

Sol.7 B

$f(x) = \{x\}$

x = 5 is minima



Sol.8 A

$f(x) = \dfrac{x - 1}{x^2} \qquad x \ge 1$

$= \left(\dfrac{1 - x}{x^2}\right) \qquad x < 1 \qquad x = 1$

$f'(x) = -\dfrac{1}{x^2} + \dfrac{2}{x^3} = 0 \quad \Rightarrow x = 2$

x = 1 and x = 2 are two critical points

Sol.9 D

$f(x) = \sin 2x - x \qquad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$f'(x) = 2\cos 2x - 1 = 0$

$\cos 2x = \dfrac{1}{2}$

$2x = \dfrac{\pi}{3}, -\dfrac{\pi}{3}$

$x = \dfrac{\pi}{6}, -\dfrac{\pi}{6}$

$f\left(\dfrac{x}{2}\right) = -\dfrac{\pi}{2} \qquad f\left(-\dfrac{x}{6}\right) = -\dfrac{1}{2} + \dfrac{\pi}{6}$

$f\left(-\dfrac{x}{2}\right) = \dfrac{x}{2} \qquad f\left(\dfrac{x}{6}\right) = \dfrac{1}{2} - \dfrac{x}{6}$

Diff $= \dfrac{x}{2} - \left(-\dfrac{x}{2}\right) = x$

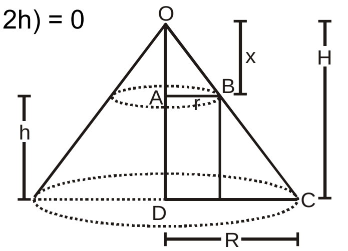
Sol.10 D

By using simllar triangle prop.

$$\frac{r}{R} = \frac{H-h}{H} \qquad ....(1)$$

$$s = 2\pi rh = 2\pi R\left(\frac{H-h}{H}\right)h$$

$$\frac{ds}{dh} = \frac{2\pi R}{H}\ (H - 2h) = 0$$



$$\Rightarrow h = \frac{H}{2}$$

$$\frac{d^2s}{dh^2} = -\frac{4\pi R}{H} < 0$$

so $\quad h = \frac{H}{2}$ is greatest

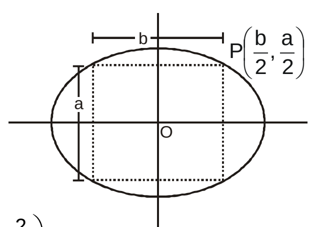
$$\frac{r}{R} = \frac{H - H/2}{H} \Rightarrow r = \frac{R}{2}$$

Sol.11 C

Area (A) = ab

Point P will lie on the given ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$



$$\frac{b^2}{16} + \frac{a^2}{9} = 4$$

A = ab

$$A^2 = a^2b^2 = a^2\left(4 - \frac{a^2}{9}\right)$$

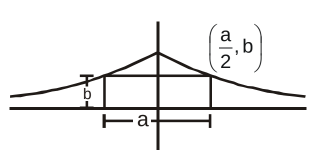
$$2A\frac{dA}{da} = 2a\left(4 - \frac{a^2}{9}\right) - a^2\left(\frac{2a}{9}\right) = 0$$

$a = 0, \quad a^2 = 18 \Rightarrow \quad a = 3\sqrt{2}$

reject $\quad \Rightarrow \quad b = 2\sqrt{8}$

Sol.12 A

$$y = e^{-x^2}$$



$b = e^{-a^2/4}$ .....(1)

A = ab

$$A = a\,e^{-a^2/4}$$

$$\frac{dA}{da} = e^{-a^2/4} + a.e^{-a^2/4}\left(-\frac{2a}{4}\right) = 0$$

$$a = \sqrt{2}$$

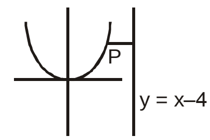
Area = $\sqrt{2}\ \ e^{-(2/4)} = \sqrt{2}\,e^{-1/2}$

Sol.13 A

Let the point on parabola P $(2t, t^2)$

$$y = \frac{x^2}{4}$$

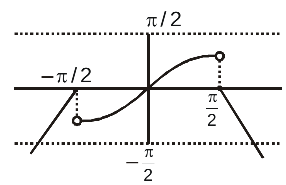
$$\frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2}\bigg|_{(2t,\,t^2)} = t$$



slope = 1 $\Rightarrow$ t = 1 so P(2, 1)

Sol.14 B

$f(x) = \tan^{-1} x\ ,\ |x| < \frac{\pi}{2}$



$\frac{\pi}{2} - |x|\ ,\ |x| \geq \frac{\pi}{2}$

$x = -\frac{\pi}{2}$ is maxima

Sol.15 D

f(1) = 1 – 1 + 10 – 5 = 5

for greatest value at x = 1

$f(1^+) \leq f(1)$ $\qquad b^2 - 2 > 0$

$-2 + \log_2 (b^2 - 2) \leq 5$; $\qquad b > \sqrt{2}$ or $b < -\sqrt{2}$

$\log_2 (b^2 - 2) \leq 7$

$b^2 - 2 \leq 2^7$

$b^2 \leq 130$

$-\sqrt{130} \leq b \leq \sqrt{130}$

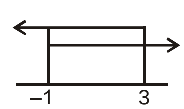
final answer $b \in [-\sqrt{130}\ , -\sqrt{2}\ ) \cup (\sqrt{2}\ , \sqrt{130}\ ]$

Sol.16 C

$f(x) = x^3 - 3px^2 + 3(p^2 - 1)x + 1$

$f'(x) = 3x^2 - 6px + 3(p^2 - 1) = 0$

	x = p – 1	&	p + 1
	p + 1 > p – 1		
so	p –1 > –2	&	p + 1 < 4
	p > –1	&	p < 3



$p \in (-1, 3)$

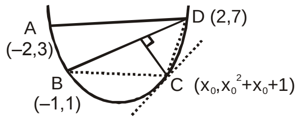
Sol.17 A

Let $y = ax^2 + bx + c$

A : $3 = 4a - 2b + c$ ....(1)

B : $1 = a - b + c$ ....(2)

C : $7 = 4a + 2b + c$ ....(3)



$b = 1 = a = c$

$y = x^2 + x + 1$

Method (1) Make determinant using area of ΔBCD then diff with respect to x_0

Method (2) Area will be maximum if tangent at C will be parallel to BD

$$\frac{dy}{dx} = 2x_0 + 1 = \left(\frac{7-1}{2+1}\right)$$

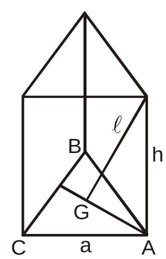
$$2x_0 + 1 = 2$$

$$x_0 = 1/2$$

$$y = \frac{1}{4} + \frac{1}{2} + 1 = \frac{1+2+4}{4} = \frac{7}{4}$$

point $\left(\frac{1}{2}, \frac{7}{4}\right)$

Sol.18 B



$$AG = \frac{2}{3} \cdot \frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$

$$\ell^2 = \frac{a^2}{3} + h^2$$

$$v(h) = 3 \cdot \frac{\sqrt{3}}{4}\, h(\ell^2 - h^2)$$

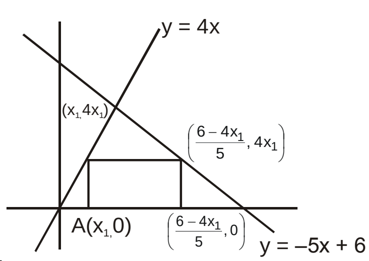
$$v'(h) = 0 \Rightarrow h = \frac{\ell}{\sqrt{3}}$$

$$v_{max} = \frac{\ell^3}{2}$$

Sol.19 C

$$A = \left(\frac{6-4x_1}{5} - x_1\right)(4x_1)$$

$$A = \left(\frac{6-9x_1}{5}\right)(4x_1)$$



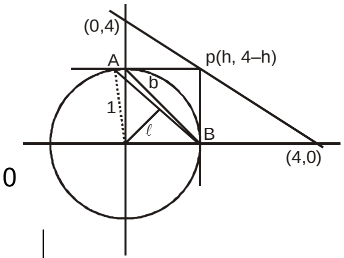
$$\frac{dA}{dx_1} = \frac{4}{5}(6 - 18x_1)$$

$$\frac{dA}{dx_1} = 0 \Rightarrow x_1 = \frac{1}{3}$$

$$A = \frac{4}{3}\left(\frac{1}{3}\right)\left(6 - 9 \times \frac{1}{3}\right) = \frac{4}{5}$$

Sol.20 C

AB = 2p



$p = \sqrt{1-\ell^2}$

AB : T = 0

$hx + (4-h)y - 1 = 0$

$$\ell = \left|\frac{0+0-1}{\sqrt{h^2 + (4-h)^2}}\right|$$

$$p = \sqrt{1 - \frac{1}{h^2 + (4-h)^2}}$$

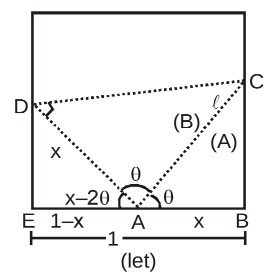
$$L = \text{length} = 2p \Rightarrow \frac{dL}{dh} = 0 \Rightarrow h = 2$$

Sol.21 B

A & B triangle are similar

$$\cos(x - 2\theta) = \frac{1-x}{x} = -\cos 2\theta$$

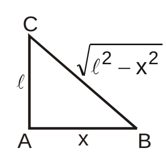
$$\cos\theta = \frac{x}{\ell}$$



$$1 - 2\cos^2\theta = \frac{1-x}{x}$$

$$1 - 2\left(\frac{x^2}{\ell^2}\right) = \frac{1-x}{x}$$

$$\Delta ABC = \frac{1}{2} x \sqrt{\ell^2 - x^2}$$



$$= \frac{1}{2}\left(\frac{x}{\ell}\right)\sqrt{1 - \left(\frac{x^2}{\ell^2}\right)}$$

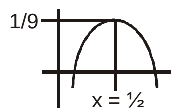
$$= \frac{1}{2}\left(\frac{x}{\ell}\right)\sqrt{1 - \frac{1}{2}\left(1 - \frac{1-x}{x}\right)} = \frac{1}{2}\left(\frac{x}{\ell}\right)\sqrt{1 + \frac{1}{2x}}$$

$$A = \frac{1}{2}\sqrt{\frac{1}{2}\left(1 - \frac{1-x}{x}\right)}\sqrt{1 + \frac{1}{2x}}$$

$$\frac{dA}{dx} = 0 \qquad \Rightarrow x = 2/3$$

Sol.22 A

$y = x - x^2$



$y' = 1 - 2x \Rightarrow x = \dfrac{1}{2}$

$y'' = -2 < 0 \qquad x = \dfrac{1}{2}$ is maxima

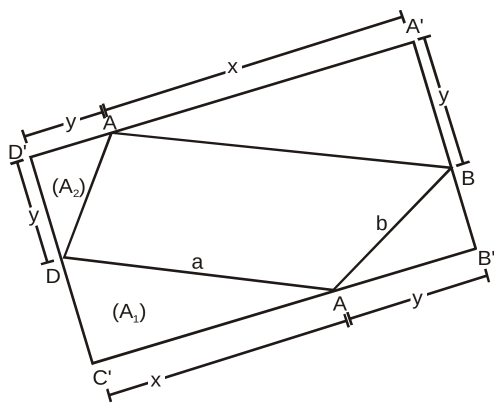
$(x_1 + x_2) - (x_1^2 + x_2^2)$

$(x_1 - x_1^2) + (x_2 - x_2^2)$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Sol.23 B

$A_1 = \dfrac{1}{2}\, x\sqrt{a^2 - x^2}$ ABCD given rectangle



$A_1^2 = \dfrac{1}{4}\, x^2(a^2 - x^2)$

$2A_1 \dfrac{dA_1}{dx} = \dfrac{a^2x}{2} - x^3 = 0 \Rightarrow x = \pm\dfrac{a}{\sqrt{2}}$

similarly $A_2 = \dfrac{1}{2}\, y\sqrt{b^2 - y^2}$

$\dfrac{dA_2}{dy} = 0 \qquad \Rightarrow y = \pm\dfrac{b}{\sqrt{2}}$

Reqd. Area = $ab + 2(A_1 + A_2)$

$= ab + x\sqrt{a^2 - x^2} + y\sqrt{b^2 - y^2}$

$= ab + \dfrac{a}{\sqrt{2}} \times \dfrac{a}{\sqrt{2}} + \dfrac{b}{\sqrt{2}} \cdot \dfrac{b}{\sqrt{2}}$

$= \dfrac{a^2}{2} + \dfrac{b^2}{2} + ab = \dfrac{(a+b)^2}{2}$

Sol.24 D

$$f(x) = 2^{x^2} + 1 + \frac{2}{2^{x^2}+1} - 2$$

f(x) will be minimum
when x = 0

$$\text{min. value} = 2 + \frac{2}{2} - 2 = 1$$

Sol.25 D

$$\frac{p+q}{2} \ge \sqrt{pq}$$

$(p + q)^2 \ge 4pq$

$p^2 + q^2 = 1$

$(p + q)^2 - 2pq = 1$

$2pq = (p + q)^2 - 1$

$4pq = 2(p + q)^2 - 2$

$2(p + q)^2 - 2 \le (p + q)^2$

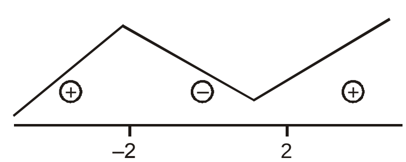
$(p + q)^2 \le 2$

$p + q \le \sqrt{2}$

Sol.26 D

$$f(x) = \frac{x}{2} + \frac{2}{x}$$

$$f'(x) = \frac{1}{2} - \frac{2}{x^2} = 0 \Rightarrow x = \pm 2$$



x = 2 is minima

Aliter Approach AM ≥ GM

$$\frac{\frac{x}{2} + \frac{2}{x}}{2} > \sqrt{\frac{x}{2} \times \frac{2}{x}}$$

$$\frac{x}{2} + \frac{2}{x} \ge 2$$

min. value = 2

It is possible only when x = 2

Sol.27 A

$$f(x) = \frac{3x^2+9x+17}{3x^2+9x+7}$$

$$f(x) = 1 + \frac{10}{3x^2+9x+7}$$

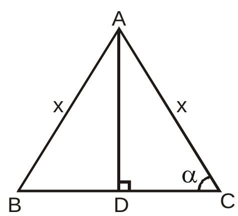
For f(x) to be maximum the quadratic expression

should get its min. value $= -\frac{0}{4a} = \frac{3}{12}$

max. value of $f(x) = 1 + \frac{10}{3/12} = 41$

Sol.28 B

In Δ ACD



$\cos\alpha = \frac{CD}{x}$

$AD = x \sin\alpha$

Area (Δ ABC) $= 2 \cdot \frac{1}{2}$ (AD) (CD)

$= (x \sin\alpha)(x \cos\alpha)$

$$A = \frac{x^2}{2}\sin 2\alpha$$

It will max. when $\sin 2\alpha = 1$

$$A = \frac{x^2}{2}$$

Sol.29 C

$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1 \qquad a > 0$

$f'(x) = 6x^2 - 18ax + 12a^2$

$= 6(x^2 - 3ax + 2a^2) = 0$

$x = 2a, a$

$f''(x) = 6(2x - 3a)\big|_{x=2a} = 6a > 0$

$f''(x) = 6(2x - 3a)\big|_{x=a} = -6a < 0$

x = 2a is minima = q

x = a is maxima = p

$p^2 = q$

$a^2 = 2a$

a = 0 (reject), a = 2

Sol.30 B

$f(x) = x^3 - 3x \qquad [0, 2]$

$f'(x) = 3x^2 - 3 = 0 \quad \Rightarrow x = \pm 1$

$f(1) = 1 - 3 = -2$

$f(-1) = -1 + 3 = 2$ (reject)

$f(0) = 0$

$f(2) = 8 - 6 = 2$

max. value = 2

Sol.31 C

$y = \frac{1}{3\sin\theta - 4\cos\theta + 7}$; $-5 < 3\sin\theta - 4\cos\theta < 5$

$$y_{min.} = \frac{1}{(3\sin\theta - 4\cos\theta + 7)_{max}}$$

$$= \frac{1}{5+7} = \frac{1}{12}$$

Sol.32 C

$f(x) = (x-p)^2 + (x-q)^2 + (x-r)^2$

$f'(x) = 2(x-p) + 2(x-q) + 2(x-r) = 0$

$\Rightarrow x = \frac{p+q+r}{3}$

$f''(x) = 2 + 2 + 2 > 0$

$x = \frac{p+q+r}{3}$ is minima

Sol.33 A

$f(x) = \cos x + \cos \sqrt{2}\, x$

f(x) will be maximum when both attains it max. value

and its only possible when x = 0

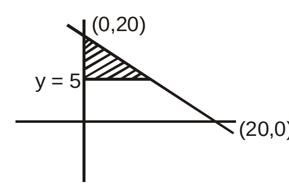
Sol.34 D

$z = 7x - 8y$

$x + y - 20 \le 0,$

$y \ge 5,\ x \ge 0$

(0, 0) $0+0-20 < 0$



Now $y \ge 5$ and

$5 \le y \le 20$; $0 \le x \le 15$

z will be min. if x contain its min. value and y contain its max. value

x = 0, y = 20 (0, 20)

Sol.35 A

maxima value of $= \cos\alpha_1 . \cos\alpha_2 . \cos\alpha_3 ... \cos\alpha_n$

Given $\cot\alpha_1 \cot\alpha_2 .... \cot\alpha_n = 1$

Possible only if $\alpha_1 = \alpha_2 = \alpha_3 .... = \alpha_n = \frac{x}{4}$

max. value $= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) ...... \left(\frac{1}{\sqrt{2}}\right)$ n times

$$= \left(\frac{1}{\sqrt{2}}\right)^n = \frac{1}{2^{n/2}}$$

Sol.36 B

Given equation $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$

Let point P ($a \cos \phi$, $2 \sin \phi$) on curve
Distance is d form (0, –2)
$z = d^2 = a^2 \cos^2 \phi + 4 (1 + \sin \phi)^2$

$$\frac{dz}{d\phi} = -2 a^2 \cos \phi \sin \phi + 8(1 + \sin \phi) \cos \phi$$

$$= (4 - a^2) \sin 2\phi + 8 \cos \phi$$

$$\frac{dz}{d\phi} = 0$$

$$\Rightarrow \sin \phi = \frac{4}{a^2 - 4} = \frac{1}{\left(\frac{a^2}{4} - 1\right)} > 1 \quad \text{as } 1 < \frac{a^2}{4} < 2$$

(reject)

$$\cos \phi = 0 \Rightarrow \phi = \frac{x}{2}$$

$$\frac{d^2z}{d\phi^2} = (4 - a^2)\, 2 \cos 2\phi - 8 \sin \phi \quad \text{if } \phi = \frac{x}{2}$$

$$= 2(a^2 - 8) < 0$$

Means $\phi = \frac{\pi}{2}$ will be maxima so point P(0, 2)

Sol.37 D

$x^3 - 3x + [a] = 0$
Let $f(x) = x^3 - 3x + [a]$
$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$
$f(1) . f(-1) < 0$ Let $t = [a]$
$(1 - 3 + t) . (-1 + 3 + t) < 0$
$(t - 2)(t + 2) < 0$
$-2 < t < 2$
$-2 < [a] < 2$
$[a] = -1, 0, 1$
$a \in [-1, 0) \cup [0, 1) \cup [1, 2)$
$a \in [-1, 2)$

Sol.38 A

$$f(x) = \sin \frac{\{x\}}{a} + \cos \frac{\{x\}}{a}$$

$$f(x) = \sqrt{2} \sin \left[\frac{\{x\}}{a} + \frac{x}{4}\right]$$

f(x) will be max. when sin take its min. value = 1

so $\frac{\{x\}}{a} + \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{\{x\}}{a} = \frac{\pi}{4} \Rightarrow \{x\} = \frac{a\pi}{4}$

$0 < \frac{a\pi}{4} < 1 \Rightarrow 0 < a < \frac{4}{\pi}$

Sol.39 B

$f(x) = x^3 + ax^2 - 9x + b$
$f'(x) = 3x^2 + 2ax - 9$
$f'(1) = 0 \Rightarrow 3 + 2a - 9 = 0 \Rightarrow a = 3$
$f'(x) = 3x^2 + 6x - 9 = 0 \Rightarrow x = 1, x = -3$
$f(1) > 0$ $f(-3) > 0$
$1 + 3 - 9 + b > 0$ $-27 + 27 + 27 + b > 0$

$\boxed{b > 5}$ $\boxed{b > 27}$

$\boxed{b > 5}$

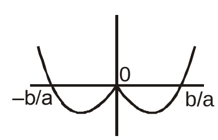
$\boxed{b = 6}$

$(a, b) = (3, 6)$

Sol.40 A,C

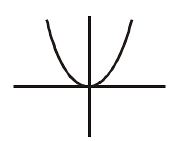
$f(x) = a|x|^2 - b|x|$
$g(x) = ax^2 - bx = ax(x - b/a)$
$x = 0, x = b/a$

(A) $a > 0, b > 0$



x = 0, maxima

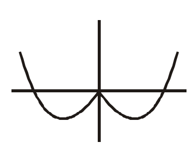
(B) $a > 0, b < 0$



x = 0, minima

(C) $a < 0, b < 0$

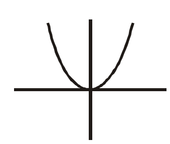
$\left(\frac{b}{a} > 0\right)$



x = 0, maxima

(D) $a < 0, b > 0$

$\frac{b}{a} < 0$

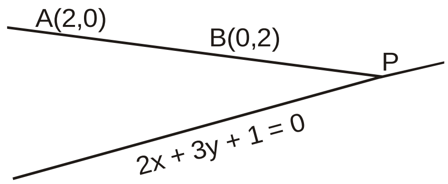


x = 0, minima

Sol.41 A

$|PA - PB|$ will be max. when P, A, B points are collinear

line AB :



$\frac{x}{2} + \frac{y}{2} = 1 \Rightarrow x + y = 2$ .....(1)

$2x + 3y + 1 = 0$ .....(2)

Point of (1) & (2) will be Point

P(7, –5)

Sol.42 C

$f(x) = 2bx^2 - x^4 - 3b$

$f'(x) = 4bx - 4x^3 = 0 \Rightarrow x = 0 \ \& \ x^2 = b$

$f''(x) = 4b - 12x^2 \big|_{x^2 = b} = 4b - 12b = -8b < 0$

so f(x) will be max at $x^2 = b$

$g(b) = 2b(b) - b^2 - 3b$

$g(b) = b^2 - 3b$

min. $g(b) = - -\frac{D}{4a} = -\frac{9}{4}$

Sol.43 A

$3x^2 - 2x^3 = \log_2(x^2 + 1) - \log_2 x$

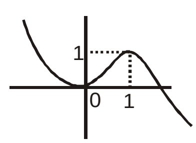
$f(x) = 3x^2 - 2x^3$

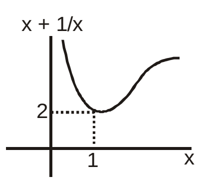
$f'(x) = 6x - 6x^2$

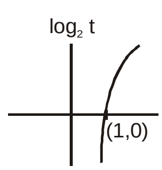
$x = 0, 1$

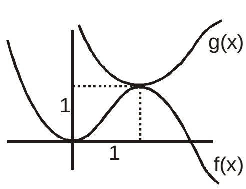
$g(x) = \log_2(x^2 + 1) - \log_2 x$

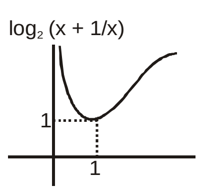
$= \log_2\left(x + \frac{1}{x}\right)$











only one solution possible

Sol.44 B

AM $\geq$ GM

$\frac{a^2x^4 + b^2y^4}{2} \geq \sqrt{a^2b^2x^4y^4}$

$\frac{c^6}{2} \geq |ab| \, x^2y^2$

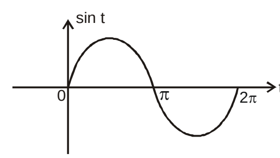
$x^2y^2 \leq \frac{c^6}{2|ab|}$

$xy \leq \frac{c^3}{\sqrt{2|ab|}}$

Sol.45 A

$f(x) = \max \ (\sin t), \ 0 < t < x, \ 0 \leq x \leq 2\pi$

$$f(x) = \begin{cases} \sin x & 0 \leq x < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \leq x < 2\pi \end{cases}$$



max. value will be 1

min. value (at x = 0) = 0

Sol.46 B

$f'(x) = \frac{1}{3(x+1)^{2/3}} - \frac{1}{3(x-1)^{2/3}}$

$f'(x) = 0 \ \Rightarrow x = 0$

$f(0) = 1 + 1 = 2$

$f(1) = 2^{1/3}$

max. value = 2

Sol.47 D

$f(x) = x^p (1 - x)^q$

$f'(x) = px^{p-1} (1 - x)^q - qx^p(1 - x)^{q-1}$

$= \frac{px^p}{x} (1 - x)^q - qx^p \frac{(1-x)^q}{(1-x)}$

$= x^p (1 - x)^q \left[\frac{p}{x} - \frac{q}{1-x}\right] = 0$

$p - px - q - x = 0 \Rightarrow x = \frac{p}{p+q}$

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Sol.48 C

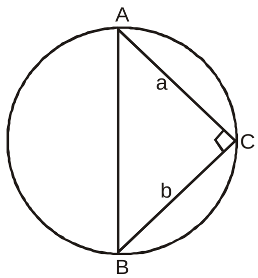
$y = -x^3 + 3x^2 + 2x - 27$

$y' = -3x^2 + 6x + 2$

$$\text{max. slope} = -\frac{D}{4a} = -\frac{(36+24)}{-12} = \frac{60}{12} = 5$$

Sol.49 A

$$S = \frac{1}{2}ab$$



$$A\ (\text{circle}) = \pi r^2$$

$$= \pi\frac{(a^2+b^2)}{4}$$

$$= \frac{\pi}{4}\left[a^2 + \frac{4S^2}{a^2}\right]$$

$AM \geq GM$

$$a^2 + \frac{4S^2}{a^2} \geq 2\sqrt{a^2 \times \frac{4S^2}{a^2}}$$

$$a^2 + \frac{4S^2}{a^2} \geq 4S$$

$$\text{Area (max.)} = \frac{\pi}{4}(4S) = \pi S$$

Sol.50 D

Let point C $(a\cos\theta, b\sin\theta)$

$$A = \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 1 & 4 & 1 \\ a\cos\theta & b\sin\theta & 1 \end{vmatrix}$$

$$A = 6 - b\sin\theta - 2a\cos\theta$$

$$\frac{dA}{d\theta} = 0 \Rightarrow \tan\theta = \frac{b}{2a} = \frac{1}{\sqrt{2}}$$

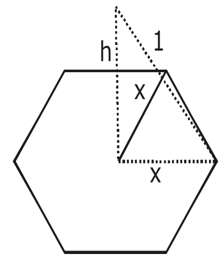
$$\Rightarrow a\cos\theta = -\sqrt{6} \text{ and } b\sin\theta = -\sqrt{6}$$

Point C $(-\sqrt{6}, -\sqrt{6})$

Sol.51 C

$x^2 + h^2 = 1$

volume



$$v = \frac{1}{3} \times \text{base} \times \text{height}$$

$$= \frac{1}{3} \times h \times 6 \times \frac{\sqrt{3}}{4}x^2$$

$$v = \frac{\sqrt{3}}{2}h(1-h^2)$$

$$\frac{dv}{dh} = 0 \Rightarrow h = \frac{1}{\sqrt{3}}$$

Sol.52 B

$f(x) = -2x^2 + 5x + 2$

$$\text{max. value} = -\frac{D}{4a} = \frac{45}{8} = 5.6$$

$\therefore$ $x \in N$, max. value will be natural number nearest natural number = 5

Sol.53 D

$$f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} \qquad \text{...(i)}$$

Replace x by 1/x

$$f\left(\frac{1}{x}\right) + f(x) = x \qquad \text{...(ii)}$$

(i) = (ii)

$$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

at $x = 1$

$f(1) + f(1) = 1 \Rightarrow f(1) = 1/2$

at $x = -1$

$$f(-1) = -\frac{1}{2}$$

$$\text{maximum value } f(1) = \frac{1}{2}$$